OUT OF PLANE BUCKLING OF
CROSS BRACING MEMBERS

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ABSTRACT

This study proposes simple technique for obtaining out of plane critical load of the compression member in cross bracing members. Nonlinear large displacement finite element analysis was performed using ANSYS program [10]. The results show that the geometrical properties of the member's cross-sections, the ratio of the tension to compression loads induced in the members and the supporting conditions at their ends affect significantly the out of plane critical load value. The intersection connection of the two members is assumed to provide full continuity. An analytical simple model is used and modified to deal with symmetrical and unsymmetrical cross bracing members with different end conditions. The results obtained using this method show good agreement with those obtained from the finite element analysis.

تقدم هذة الدراسة طريقة بسيطة لحساب حمل الأنبعاج لأعضاء التربيط المتقاطعة عند أنبعاجها فى مستوى اعمودى على مستوى تقاطعها. تم أستخدام طريقة العناصر المحدودة اللا خطية و التى تتضمن تأثير الأزاحات لتمثيل سلوك أعضاء التربيط عند أنبعاجها. النتائج المستخرجة توضح أن الخواص الهندسية لقطاعات أعضاء التربيط و نسبة قوة الشد التى قد تنتج فى أحد أعضاء التربيط الى قوة الضغط فى العضو الأخر بالأضافة الى نوع الركائز عند أطراف الأعضاء تؤثر تأثيرا كبيرا على قيمة حمل الأنبعاج. تم أفتراض أن الوصلة بين أعضاء التربيط مستمرة. تم أستخدام نموذج بسيط لحساب حمل الأنبعاج و تعديلة لأمكانية أستخدامة لأعضاء التربيط المتماثله و غير المتماثلة و التى لها ركائز مختلفة عند أطرافها. النتائج المستخرجة من هذة الطريقة تتوافق مع تلك المستخرجة من طريقة العناصر المحدودة.

INTRODUCTION

Diagonal cross bracing members are commonly used in structural steel works to resist horizontal loads and/or to reduce effective unrestrained length of compression members. This would subject one member to tension load and the other one to compression. In design practice, the compression member is commonly assumed not to be effective. Only the tension member is designed. The compression member is taken typical to the tension member. The Egyptian code for steel construction and design ECP [1] prohibit the use of rods and cables in bracing systems. For members in buildings, designed on basis of tension, the ECP [1] and the American specification AISC [2] specify that the maximum slenderness ratio λ should not exceed 300.

Diagonal cross bracing members are repeated many times in steel structures. Including the contribution of the compression diagonal would produce economical design. In seismic areas, it is important to predict in which plane the system will buckle. The end connection should be detailed to permit ductile rotation in the buckling plane. If buckling occurs in the perpendicular plane, the connection may fracture prematurely. The compression diagonal should be designed against in plane and out of plane buckling.

Several studies in the literature show that the diagonal tension member provides degree of restraint to the compression member against out of plane buckling. Different expressions have been derived. Timoshinko and Gere [3] used differential equations to find the relationship between the critical load and brace stiffness for a column with mid height brace. They showed that there is a limit spring stiffness above which the spring would behave as if it were a hinged support. Winter [4] proposed simple model to find the value of this stiffness. Stoman [5], [6] and [7] employed Raleigh − Ritz method of stationary potential energy to formulate closed form stability criteria for evaluating the transverse stiffness provided by the tension brace. Picard [8] concluded that the effective length of the compression diagonal is 0.5 times the diagonal length for both out of plane and in plane buckling. The same conclusion was drawn by El Tayem et. al. [9]. Most of these studies dealt with the problem as a two dimensional problem. The members are assumed to have identical length, cross sections and material properties. Further, one diagonal is under tension while the other one is subjected to compression. The intersection of the two members is at half-length

and provides full continuity. The supporting conditions of each member are hinged at one end and roller at the other one.

This study investigates the parameters that affect the out of plane critical load P_C of the compression member in cross bracing members. Buckling is assumed to occur about one of the cross section principles' axes. Bracing members of single angles are not included in this study. A three-dimensional finite element analysis was performed. An analytical simple model is used and modified to deal with cross bracing members when they are symmetrical and unsymmetrical and having different end conditions.

FINITE ELEMENT ANALYSIS

(i) Mesh

Buckling behavior of cross bracing members is modeled using nonlinear large displacement elastic finite element analysis. The ANSYS program [10] was used to perform the analysis. Figure 1 shows two crossing members, B and C. Each member was modeled using 20 uniaxial beam elements with tension, compression, torsion and bending capabilities. The element has 6 degrees of freedom at each node: translations in the directions of and rotations about the node X, Y and Z-axes. Large deflection capabilities of the element are activated. Different supporting conditions were considered. The material behavior was modeled to be elastic. The modulus of elasticity E value is taken equal to 2100 t/cm^2 according to the ECP [1]. Concentrated compression load P was applied at point C_1, figure 1. In some cases, tension load T was applied as well at point B_1. This is to simulate the behavior when compression force induces in one member and tension force induces in the other one.

(ii) Modeling of buckling

Two cases of initial out of straightness were considered for the mesh of the compression member. First, the imperfection displacement field was given a half sine wave along member C in the Y - Z plane. The buckling in this mode represents the case of a hinged – hinged column and named mode 1 of buckling. Second, a full sine wave was superimposed along member C. The buckling in this mode represents the case of a hinged – hinged column supported at its middle by a hinged support and named mode 2 of buckling. The maximum initial imperfection of the mesh was made equal to 1 / 500 of the member length L. This technique is used in the literature and

known as the seeding technique [11] and [12]. Incremental nonlinear solutions for the two cases were obtained and evaluated for buckling. This is to guarantee that the buckling occurred is the first buckling mode, which happens in practice. The critical load P_C is defined to occur at a load corresponding to very large deflection at which the tangentt of the load – deflection relationship is smaller than a specified tolerance. This tolerance was set to 2×10^{-3} KN/mm. The Newton-Raphson technique was used for equilibrium convergence with a tolerance limit of 0.01. The convergence criteria were based on checking forces [10]. The model was verified against standard cases.

PARAMETRIC STUDY

The parameters considered are the geometrical properties of the members' cross-sections, ratio of the tension to compression loads induced in the members and the supporting conditions. The connection at the intersection of the two members is at their half-length and assumed to provide full continuity to both of them. The effect of this continuity is also examined.

1- Cross Section Geometrical Properties

Members B and C in figure 1 were given the same length and material properties.

Different values for the moments of inertia of member C about the X and Y-axes, $(I_X)_C$ and $(I_Y)_C$, and those of member B about the Y and Z-axes, $(I_Y)_B$ and $(I_Z)_B$, were considered. Hinged supports were provided at points B_2 and C_2. Roller supports are provided at points B_1 and C_1 that allow transition in the directions of the X and Z axes respectively. Rotations about the X, Y and Z-axes were allowed at all the supports.

(i) Effect of $(I_Z)_B$

The moments of inertia $(I_X)_C$, $(I_Y)_C$ and $(I_Y)_B$ were made constant and given the same value. The value of $(I_Z)_B$ was varied. The critical load P_C values corresponding to the out of plane buckling were obtained. The results are presented in figure 2 in terms of the ratios $(I_Z)_B / (I_X)_C$ and P_C / P_E where P_E is the Eular load of member C; i.e. $P_E = \pi^2$ E $(I_X)_C / L^2$. The results show that the values of P_C / P_E are linearly proportional to $(I_Z)_B / (I_X)_C$ to a certain limit after which the value of P_C / P_E become nearly constant. At $(I_Z)_B / (I_X)_C = 0.1$, the value of P_C is equal 108% of P_E . This means that member B provides relatively low degree of support to member C against out of plane buckling.

For $(I_Z)_B$ / $(I_X)_C$ ranging between 3.4 to 4, the values of P_C / P_E are found to vary between 3.92 and 3.95. The buckling in this range occurred in mode 2. Member B restrains member C as if it were a hinged support. Increasing the value of $(I_Z)_B$ further did not cause significant change in the critical load and buckling mode. By reviewing the values of the reactions at the different supporting points for the different cases considered, the following notes were noticed. At buckling, the reactions in the Z-axis direction at B_1 and B_2 were nearly negligible. Almost all the applied compression load was transmitted to C_2. The reaction in the X-axis direction at point B_2 did not exceed 0.6% of P_C. However, the reactions at points B_1 and B_2 in the Y-axis direction were nearly equal and ranging between 1.5% and 9.4% of P_C. Their values were proportional to the values of $(I_Z)_B$. This is explained as follows. Increasing the value of $(I_Z)_B$ would increase the flexural stiffness of member B about the Z axis. This would require more force in the Y- axis direction to displace member B and hence allow member C to buckle in Y-Z plane.

(ii) Effect of $(I_Y)_B$

The moments of inertia $(I_X)_C$,$(I_Y)_C$ and $(I_Z)_B$ were made constant and given the same value; i.e. $(I_Y)_C$ /$(I_X)_C$ = $(I_Z)_B$ /$(I_X)_C$ =1.0. The value of $(I_Y)_B$ was varied. The results obtained are presented in figure 3. The ratio of P_C / P_E is found to be proportional to $(I_Y)_B$ / $(I_X)_C$. Unlike $(I_Z)_B$, increasing the value of $(I_Y)_B$ / $(I_X)_C$ from 0.02 to 3 caused limited increase in P_C /P_E that did not exceed 7.5%. It is noted that, when applying compression load P on member C, the joint at the intersection of members B and C displace down wards in the direction of the Z-axis. Part of this load is transmitted through this joint to member B, and hence to the supports at B1 and B2 in turn. The results show that the value of this part of the load is proportionally affected by the value of $(I_Y)_B$. At buckling, its value is nearly negligible in comparison to P_C.

(iii) Effect of $(I_Y)_C$

The cross section of member C was modeled having the geometrical properties of rectangular hollow section RHS 203*102*4.8 complying with the Canadian Standard Specification CSA [13]. In this case the value of $(I_Y)_C$ /$(I_X)_C$ = 2.92. The value of $(I_Z)_B$ was made equal to $(I_X)_C$ while the value of $(I_Y)_B$ was varied. The results obtained are found typical to those of figure 3 for $(I_Y)_C$ / $(I_X)_C$ = 1. The change in $(I_Y)_C$ has no effect on the out of plane critical load.

2- Effect of Tension Load

Members B and C of figure 1 were modeled having the same cross-section, length, and material properties. The moments of inertia of members B and C cross sections were given the same values; i.e. $(I_Y)_C$ / $(I_X)_C = (I_Y)_B$ / $(I_Z)_B$, and hence $(I_Z)_B$ / $(I_X)_C = (I_Y)_B$ / $(I_X)_C = 1$. The restraining conditions are as in figure 1. Compression and tension loads were applied at C_1 and B_1 respectively. Different values of T / P ratio were considered. For each case, the values of T and P were increased, but keeping their ratio T / P constant. The values of P_C were obtained and presented in terms of the ratio P_C / P_E, figure 4. The results in general show that P_C / P_E is bilinear proportional to the ratio T / P to certain limit after which the value of P_C / P_E is constant. When no tension load is applied, the value of P_C / $P_E = 2.01$. The value of P_C / $P_E = 4.0$ when T / P equals 0.628, figure 4. In this case, member B restrains member C against out of plane buckling as it were a hinged support. Picard et al [8] found analytically that P_C / P_E would equal 4.0 when T / P = 0.625 and increasing T /P further would not cause any increase in P_C / P_E. The finite element results however show different behavior. Increasing T / P ratio more than 0.628 elevated P_C / P_E value. This is valid up to T / P = 0.8 after which P_C / P_E value is constant. It should be noted that the inclination of part a b of the relation in figure 4 is different to that of b c.

3 - Effect of Supporting Conditions

Members B and C in figure 1 were modeled having the same length and material properties. They were given cross section of RHS 203*102*4.8 complying with CSA [13]. Six cases of different out of plane supporting conditions; i.e. in the Y-Z plane, were considered. Table 1 shows the obtained values of P_C / P_E ratio. The results show that the type of supporting conditions at the members' ends affects the critical load significantly. Increasing the fixity degree at the ends of member C is more effective than doing this to member B. When changing the supporting conditions of members B and C in case 1 to those in case 6, the critical load increased by nearly 4 times. In this case, member B provided support as if it were a hinge and each part of member C behaved as if it were hinged-fixed column.

Table 1 Values of P_c / P_E at different supporting conditions

Case	Properties of members B & C	Supporting conditions*				P_c / P_E		Ratio**
		Member C		Member B		F.E. analysis	Analytical model	
		C_1	C_2	B_1	B_2			
1	the same length, material properties cross section of RHS 203*102*48 $(I_Y)_B/(I_Z)_C=(I_Y)_C/(I_X)_C=2.92$ $(I_Z)_B/(I_X)_C=(I_Y)_B/(I_Y)_C=1$ NO tension is induced in B	R	H	R	H	2.06	1.91	92.7%
2				R	F	3.22	3.04	94.4%
3				FR	F	3.94	4.0	101.5%
4		FR	F	R	H	5.14	4.97	96.6%
5				R	F	6.47	6.16	95.2%
6				FR	F	8.05	7.87	97.7%

Notes
* Symbols used for supporting conditions means:
R = roller support that allows transition in the direction of the member length,
H = hinged support, F = fixed support and FR = fixed support which allows transition in the direction of the member length.
** Ratio = % of simple model results to the F. E. results

4 - Intersection Connection

In practice, cross bracing members are usually made co-planner. One member is interrupted and the other one is continuous. It is usual practice to connect the interrupted member to the continuous one by means of gusset plate connection. The finite element analysis was used to model case 1 of table 1 when member B is interrupted and connected to member C by hinges as shown in figure 5. The results show that member B in this case does not provide any degree of restraining to C against out of plane buckling and the value of $P_C / P_E = 1.0$. The study in reference [14] considered the cases of semi-rigid intersection connection for cross bracing members with pinned end connections.

ANALYTICAL MODEL

Cross bracing members are modeled as follows. The compression member is supported by a spring at the two members intersection. Winter [4] modified this model by introducing a fictitious hinge as shown in figure 6. This is to find the spring stiffness value after which the spring would restrain the compression member against buckling as if it were a hinged support. The spring is assumed to be elastic. At buckling, the equilibrium at the hinge O is given as follows:

$$P_C \ \Delta \ = (K \ \Delta / 2) * (L / 2) \qquad (1)$$

Where P_C is the critical load and K is the transitional stiffness of the spring. At buckling, each part of the compression member would buckle individually and the critical load of the system would equal:

$$P_C = \pi^2 \, E \, I / (\, L \, / \, 2 \,)^2 = 4 \, P_E \qquad (2)$$

Where P_E is the Eular load of the compression member. By substituting P_C of equation 2 into equation 1, the spring would behave as if it were a hinged support when:

$$K = 16 \, P_E \, / \, L \qquad (3)$$

For the cross bracing members B and C in figure 1, K represents the transitional stiffness of member B that provide restraining to member C against out of plane buckling. From the finite element results, the values of F_y and δ_y are related by equation 4. The symbols F_y and δ_y are used for the force induced and the deflection occurred at members B and C intersection in the Y-axis direction.

$$\delta_y = F_y \, L^3 \, / \, 48 \, E \, (\, I_Z \,)_B \qquad (4)$$

Equation 4 can be rearranged as follows:

$$K = F_y \, / \, \delta_y = S_B \, E \, (\, I_Z \,)_B \, / \, L^3 \qquad (5)$$

Where $S_B = 48$. Equation 5 is substituted into equation 3. In figure 7, equation 3 is represented by the dashed line o e f in terms of the ratios $P_C \, / \, P_E$ and $(I_Z)_B \, / \, (I_X)_C$. P_E in this case is the Eular load for out of plane buckling of member C and taken as follows:

$$P_{E =} \pi^2 \, E \, (\, I_X \,)_C \, / \, L^2 \qquad (6)$$

The results show that member B would behave as if it were a hinged support at $(\, I_Z \,)_B \, / \, (\, I_X \,)_C = 3.29$. This relation is modified by the line d e f for the original proposed model; i.e. with out the fictitious hinge. For comparison, the finite element results of

figure 2 are superimposed on figure 7. The two relations are coinciding up to $(I_Z)_B$ / $(I_X)_C = 1.85$ and then deviates. The maximum difference in P_C / P_E values obtained from the two analyses equals 4% at $(I_Z)_B$ / $(I_X)_C = 3.29$.

EVALUATION OF OUT OF PLANE Critical LOAD

(i) Symmetrical members

Figure 8 can be used for the evaluation of out of plane critical load for cross bracing members as members B and C in figure 1. The relationship d e f of figure 7 is implemented for the cases when no tension is considered. The finite element results at different values of T/P_C are superimposed. Member B is assumed to provide a hinged support at $T / P_C = 0.628$ and the increase in T / P_C is assumed to cause no increase in P_C/P_E. Figure 9 is used when members B and C have fixed supports at their ends as shown in the figure. Winter model [4] is not valid for this case. This relation is obtained as follows. When member C is supported only at its ends by fixed supports, its critical load $P_C = 4 P_E$. When member B is considered, it is modeled as an elastic spring. This spring would restrain member C against buckling as if it were a hinged support when:

$$K = 21 P_E / L \tag{7}$$

And the critical load in this case would equal:

$$P_C = 8.184 P_E \tag{8}$$

These values are obtained using the stability functions Φ and Ψ in reference [3]. By equating equation 7 to 5 and using $S_B = 192$, member B would restrain C as if it were a hinged support at $(I_Z)_B$ / $(I_X)_C = 1.08$.

(ii) Unsymmetrical members

Figures 8 and 9 can still be used when cross bracing members are not symmetrical; i.e. having different lengths, supporting conditions and/or cross sections. In this case, a fictitious member having supporting conditions and length typical to those of member C is used instead of member B. The transitional stiffness provided by that member should equal that of member B and calculated as follows:

$$S_B \, E \, (I_Z)_B \, / \, L_B{}^3 = S_F \, E \, (I_Z)_F \, / \, L_C{}^3 \qquad\qquad (9)$$

The symbols B, C and F are used for members B, C and the fictitious one respectively. The value of S_F would depend on the supporting conditions of member C. S_F equals 48 and 192 when dealing with figures 8 and 9 respectively. Similarly, the value of S_B would equal 48, 107.3 or 192 when the supporting conditions of member B are hinged-roller, fixed-roller or fixed-fixed respectively. The value of $(I_Z)_F$ is obtained in terms of $(I_Z)_B$. The ratio of $(I_Z)_F \, / \, (I_X)_C$ is calculated and used instead of $(I_Z)_B \, / \, (I_X)_C$ to get the value of P_C/P_E from figures 8 or 9. It should be noted that this method do not include the effect of $(I_Y)_B \, / \, (I_X)_C$. Figures 8 and 9 are used to obtain the values of P_C/P_E for cases 1 to 6 of table 1. The results are presented in table 1 and show good agreement.

PRACTICAL CONSIDRATIONS

In practice, designers normally use typical cross sections for diagonal cross bracing members. In this case, the values of $(I_Z)_B \, / \, (I_X)_C = (I_Y)_B \, /(I_Y)_C = 1$. The value of $(I_Y)_B \, / (I_X)_C = (I_Y)_B \, / \, (I_Z)_B$ which is the ratio of the moments of inertia of the member's cross section about its principle axes. This ratio equals unity for circular and square hollow sections. By reviewing the Canadian specification CSA [13], it is found that $(I_Y)_B \, / (I_Z)_B$ ranging between 1.5 and 3.1 for rectangular hollow sections. For two angles back to back, the value of $(I_Y)_B \, / \, (I_Z)_B$ would depend on the thickness of the gusset plate and either the short or the long leg is parallel to the Y axis. The values of $(I_Y)_B \, / (I_Z)_B$ are calculated using the data provided in the CSA [13] considering the global coordinate system in figure 1. The values of $(I_Y)_B \, / \, (I_Z)_B$ are found ranging between 0.125 to 0.47 for equal leg angles. For unequal leg angles, $(I_Y)_B \, / \, (I_Z)_B$ values are ranging between 0.46 to1.25 when the short legs are parallel to the Y axis and 0.09 to 0.2 when the long legs are parallel to the Y axis. This in turn limits the effect of $(I_Y)_B \, / (I_X)_C$ on the value of $P_C \, /P_E$.

CONCLUSIONS

Nonlinear large displacement finite element analysis was used to model the buckling behavior of cross bracing members using ANSYS program. One member is subjected to compression load and the other one to tension load in some cases, members B and

C in figure 1. The results show that member B provides degree of restrain to member C against out of plane buckling. The value of $(I_Z)_B$ / $(I_X)_C$ effects significantly this degree of restraining. At $(I_Z)_B$ / $(I_X)_C = 3.4$, member B would restrain C against out of plane buckling as if it were a hinged support. Another effective parameter is the tension load induced in B. At $T/P_C = 0.628$, the value of P_C / $P_E = 4.0$. The supporting conditions at the ends of members C and B are other parameters that affect the out of plane critical load, table1. Further, an analytical simple model is implemented and developed to evaluate the out of plane critical load. Figures 8 and 9 can be used for symmetrical and unsymmetrical cross bracing members. The results obtained are compared to those of the finite element. The results show good agreement.

NOMENCLATURE

E	modulus of elasticity
F_Y	force in the direction of the Y axis
$(I_N)_M$	moment of inertia of member M about N-N axis
K	spring transitional stiffness
L	member length
P	compression load
P_C	critical load
P_E	Eular load
Sn	numerical factor of member N
T	tension load
Δ	deflection at mid length of member
Δ_b	deflection at buckling
δ_y	deflection in the direction of the Y-axis

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